

Interpretation of neutrino oscillations based on new physics in the infrared

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ABSTRACT: An interpretation of neutrino oscillations based on a modification of relativistic quantum field theory at low energies, without the need to introduce a neutrino mass, is seen to be compatible with all observations.

KEYWORDS: Beyond Standard Model, Neutrino Physics, Space-Time Symmetries.

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1. Introduction

The now well-established observation of deficit of solar neutrinos, atmospheric neutrinos, and neutrinos from reactors and accelerators finds a coherent interpretation in terms of neutrino oscillations between three neutrino flavors of different masses [1]. In the minimal standard model (SM), and in contrast with the rest of the matter particles, the neutrino is assumed to be a zero mass, left-handed fermion. Therefore neutrino oscillations is our first glimpse of physics beyond the SM.

Massive neutrinos are introduced in extensions of the SM which normally invoke new physics at high energies. In particular, one can consider a Majorana mass term for the neutrino, generated by a five-dimensional operator in the SM Lagrangian which would be suppressed by the inverse of a certain high-energy scale. Another possibility is to enlarge the field content of the SM with a right-handed neutrino, which allows mass to be generated by the usual Higgs mechanism. One has to account however for the smallness of the neutrino mass, which is achieved by the see-saw mechanism [2], again invoking a grand-unification scale.

The presence of new physics at high energies has been explored in several attempts to find alternatives to the standard neutrino oscillation mechanism. This new physics might include Lorentz and/or CPT violations. These two low-energy symmetries are being questioned at very high energies in the framework of quantum gravity and string theory developments [3], and in fact simple models with Lorentz and/or CPT violations are able to generate neutrino oscillations, even for massless neutrinos [4]. Some of them are considered in the context of the Standard Model Extension (SME) [5], which is the most general framework for studying Lorentz and CPT violations in effective field theories.

However, all these alternative mechanisms involve new energy dependencies of the oscillations which are in general disfavored over the standard oscillation mechanism by experimental data, which also put strong bounds on the contribution of new physics to this

phenomenon [6]. This seems to indicate that our understanding of neutrino oscillations as driven by mass differences between neutrino flavors is indeed correct.

In this letter we want to argue that this might not be the case. We will present an example of new physics to the SM able to generate neutrino oscillations and which is essentially different from previously considered models in one or several of the following aspects: it does not necessarily add new fields to those present in the SM, it may be completely indistinguishable from the standard oscillation mechanism in the energy ranges where the phenomenon has been studied (for neutrinos of medium and high energies), so that automatically satisfies all constraints which are already fulfilled by the standard mechanism, and finally, it predicts new physics in the infrared, so that future neutrino low energy experiments could distinguish this mechanism from the standard one.

Our example will be based on the so-called theory of noncommutative quantum fields, which has recently been proposed as a specific scheme going beyond quantum field theory [7–9]. The consequences for neutrino oscillations of a simple model with modified anticommutators for the neutrino fields, which can be identified as an example of a SME in the neutrino sector, has very recently been explored in ref. [10]. We will see however that it is possible to introduce a generalization of the anticommutation relations of fields in a more general way than that studied in ref. [10], going beyond the effective field theory framework of the SME, which is the key to reproduce the oscillation results without the need to introduce a neutrino mass, and with new consequences at infrared energies.

2. Noncanonical fields and neutrino oscillations

The theory of noncommutative fields was first considered in refs. [7, 8]. It is an extension of the usual canonical quantum field theory in which the procedure of quantification of a classical field theory is changed in the following way: the quantum Hamiltonian remains the same as the classical Hamiltonian, but the canonical commutation relations between fields are modified. In the case of the scalar complex field this modification leads to the introduction of two new energy scales (one infrared or low-energy scale, and another one ultraviolet or high-energy scale), together with new observable effects resulting from the modification of the dispersion relation of the elementary excitations of the fields [8]. If one is far away from any of these two scales the theory approaches the canonical relativistic quantum field theory with corrections involving Lorentz invariance violations which can be expanded in powers of the ratios of the infrared scale over the energy and the energy over the ultraviolet scale. By an appropriate choice of the two new energy scales one can make the departures from the relativistic theory arbitrarily small in a certain energy domain.

We will now explore the relevance of the extension of relativistic quantum field theory based on noncanonical fields in neutrino oscillations. In particular, we will show that it is possible to obtain oscillations with the observed experimental properties just by considering a modification of the anticommutators of the fields appearing in the SM, without the need to introduce a right-handed neutrino or a mass for this particle.

With the left-handed lepton fields of the SM

$$\Psi_{L\alpha} = \begin{pmatrix} \nu_\alpha \\ l_\alpha \end{pmatrix}_L, \quad (2.1)$$

where α runs the flavor indices, ($\alpha = e, \mu, \tau$), the simplest way to consider an analog of the extension of the canonical quantum field theory for a complex scalar field proposed in refs. [7, 8] is to introduce the modified anticommutation relations

$$\{\nu_{L\alpha}(\mathbf{x}), \nu_{L\beta}^\dagger(\mathbf{y})\} = \{l_{L\alpha}(\mathbf{x}), l_{L\beta}^\dagger(\mathbf{y})\} = [\delta_{\alpha\beta} + A_{\alpha\beta}] \delta^3(\mathbf{x} - \mathbf{y}). \quad (2.2)$$

A particular choice for the matrix $A_{\alpha\beta}$ in flavor space which parametrizes the departure from the canonical anticommutators corresponds to the new mechanism for neutrino oscillations proposed in ref. [10] which, however, is not compatible with the energy dependence of the experimental data.

In order to reproduce the observed properties of neutrino oscillations [1] one has to go beyond this extension and consider an anticommutator between fields at different points. This can be made compatible with rotational and translational invariance and with $SU(2) \times U(1)_Y$ gauge symmetry by making use of the Higgs field

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad \tilde{\Phi} = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}. \quad (2.3)$$

The modified anticommutators of the left-handed lepton fields that we consider in this work are

$$\{\Psi_{L\alpha}(\mathbf{x}), (\Psi_{L\beta})^\dagger(\mathbf{y})\} = \delta_{\alpha\beta} \delta^3(\mathbf{x} - \mathbf{y}) + \tilde{\Phi}(\mathbf{x}) \tilde{\Phi}^\dagger(\mathbf{y}) B_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|), \quad (2.4)$$

where $B_{\alpha\beta}$ are now functions of $|\mathbf{x} - \mathbf{y}|$ instead of constants. Note that eq. (2.4) is compatible with gauge invariance since $\tilde{\Phi}(\mathbf{x})$ has the same $SU(2) \times U(1)_Y$ quantum numbers as $\Psi_{L\alpha}(\mathbf{x})$.

After introduction of spontaneous symmetry breaking ($\langle \varphi^0 \rangle = v/\sqrt{2}$), and neglecting effects coming from the fluctuation of the scalar field which surely is a good approximation for neutrino oscillations, the only anticommutators that are changed are those of the neutrino fields

$$\{\nu_{L\alpha}(\mathbf{x}), \nu_{L\beta}^\dagger(\mathbf{y})\} = \delta^3(\mathbf{x} - \mathbf{y}) \delta_{\alpha\beta} + C_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|), \quad (2.5)$$

where

$$C_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|) = \frac{v^2}{2} B_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|). \quad (2.6)$$

One can suspect that the new anticommutation relations eq. (2.5) introduce a source of mixing between flavors that will affect neutrino oscillations. We will see in the next section that this is indeed the case.

3. Solution of the free theory

In order to study the neutrino oscillations induced by the modified anticommutators of fields in the neutrino sector, one needs to solve the free theory given by the Hamiltonian

$$H = \sum_{\alpha} \int d^3 \mathbf{x} \left[i \nu_{L\alpha}^{\dagger} (\boldsymbol{\sigma} \cdot \nabla) \nu_{L\alpha} \right] \quad (3.1)$$

(where $\boldsymbol{\sigma}$ are the 2×2 Pauli matrices), and the anticommutation relations showed in eq. (2.5).

Let us introduce a plane wave expansion for the neutrino field

$$\nu_{L\alpha}(\mathbf{x}) = \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2p}} \sum_i \left[b_i(\mathbf{p}) u_{L\alpha}^i(\mathbf{p}) e^{i\mathbf{p} \cdot \mathbf{x}} + d_i^{\dagger}(\mathbf{p}) v_{L\alpha}^i(\mathbf{p}) e^{-i\mathbf{p} \cdot \mathbf{x}} \right], \quad (3.2)$$

where $p = |\mathbf{p}|$, and $(b_i(\mathbf{p}), d_i^{\dagger}(\mathbf{p}))$ are the annihilation and creation operators of three types of particles and antiparticles (expressed by subindex i) with momentum \mathbf{p} . We now use the following *ansatz* for the expression of the Hamiltonian (3.1) as a function of the creation-annihilation operators:

$$H = \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \sum_i \left[E_i(p) b_i^{\dagger}(\mathbf{p}) b_i(\mathbf{p}) + \bar{E}_i(p) d_i^{\dagger}(\mathbf{p}) d_i(\mathbf{p}) \right]. \quad (3.3)$$

This corresponds to the assumption that the free theory describes a system of three types of free particles and antiparticles for each value of the momentum, with energies $E_i(p), \bar{E}_i(p)$, respectively.

Now, computing $[H, \nu_{L\alpha}]$ by two procedures: firstly by using eq. (3.1) for the Hamiltonian and the anticommutators (2.5), and secondly, by using the expressions (3.2) and (3.3), and equalling both results, one obtains the following simple result for the energies and the coefficients in the plane wave expansion of the field:

$$E_i(p) = \bar{E}_i(p) = p [1 + \tilde{c}_i(p)], \quad (3.4)$$

$$u_{L\alpha}^i(\mathbf{p}) = v_{L\alpha}^i(\mathbf{p}) = e_{\alpha}^i(p) \chi^i(\mathbf{p}), \quad (3.5)$$

where $\chi^i(\mathbf{p})$ is the two component spinor solution of the equation

$$(\boldsymbol{\sigma} \cdot \mathbf{p}) \chi^i(\mathbf{p}) = -p \chi^i(\mathbf{p}) \quad (3.6)$$

with the normalization condition

$$\chi^{i\dagger}(\mathbf{p}) \chi^i(\mathbf{p}) = 2E_i(p), \quad (3.7)$$

$\tilde{c}_i(p)$ are the three eigenvalues of $\tilde{C}_{\alpha\beta}(p)$, the Fourier transform of $C_{\alpha\beta}(|\mathbf{x} - \mathbf{y}|)$ in eq. (2.5), and $e_{\alpha}^i(p)$ are the components of the normalized eigenvectors of $\tilde{C}_{\alpha\beta}(p)$.

From eq. (3.4), we see that the model presented here contains violation of Lorentz invariance, but preserves *CPT* symmetry.

4. New IR physics and neutrino oscillations

Since in the free theory solution there are three types of particles and antiparticles with different energies, and a mixing of creation and annihilation operators of different kinds of particle-antiparticle in the expression of each field, it is clear that the nonvanishing anticommutators of different fields will produce neutrino oscillations, even for massless neutrinos. This observation was already present in ref. [10]. The probability of conversion of a neutrino of flavor α produced at $t = 0$ to a neutrino of flavor β , detected at time t , can be directly read from the propagator of the neutrino field (3.2). This probability can be written as

$$\mathcal{P}(\nu_\alpha(0) \rightarrow \nu_\beta(t)) = \left| \sum_i e_\alpha^i(p)^* e_\beta^i(p) e^{-i E_i(p)t} \right|^2. \quad (4.1)$$

This is the standard result for the oscillation between three states with the unitary mixing matrix elements U_α^i replaced by the coefficients $e_\alpha^i(p)$ of the plane wave expansion of the noncanonical neutrino fields and the energy of a relativistic particle $\sqrt{p^2 + m_i^2}$ replaced by the energy $E_i(p)$ of the particle created by the noncanonical neutrino fields.

Let us now make the assumption that the modification of the anticommutators is a footprint of new physics at low energies, parametrized by an infrared scale λ . Although the introduction of corrections to a quantum field theory parametrized by a low-energy scale has not been so well explored in the literature as the corrections produced by ultraviolet cutoffs, there are several phenomenological and theoretical reasons that have recently lead to think on the necessity to incorporate a new IR scale to our theories [11–14].

If the modifications of the anticommutation relations are parametrized by an infrared scale λ then it is reasonable to assume an expansion in powers of λ^2/p^2 so that

$$\tilde{C}_{\alpha\beta}(p) \approx \tilde{C}_{\alpha\beta}^{(1)} \frac{\lambda^2}{p^2} \text{ for } p^2 \gg \lambda^2, \quad (4.2)$$

and then

$$\tilde{c}_i(p) \approx \tilde{c}_i^{(1)} \frac{\lambda^2}{p^2}, \quad e_\alpha^i(p) \approx e_\alpha^{i(1)}, \quad (4.3)$$

where $e_\alpha^{i(1)}$ ($\tilde{c}_i^{(1)}$) are eigenvectors (eigenvalues) of $\tilde{C}_{\alpha\beta}^{(1)}$, independent of p .

But in this approximation, the description of neutrino oscillations produced by the new physics is completely undistinguishable from the conventional description based on mass differences (Δm_{ij}^2) with a mixing matrix (U_α^i) between flavor and mass eigenstates, just by making the correspondence

$$\Delta m_{ij}^2 = 2(\tilde{c}_i^{(1)} - \tilde{c}_j^{(1)})\lambda^2, \quad U_\alpha^i = e_\alpha^{i(1)}. \quad (4.4)$$

One should note that when $\lambda \neq 0$ the different energies for different states select a basis in the Fock space and the mixing of Fock space operators in the fermionic fields is unavoidable. It is only when one considers the energy splitting of the different particles that one has a physical consequence of the mixing of different creation-annihilation Fock space operators in each fermionic field. On the other hand, in the case $\lambda = 0$ (corresponding

to unmodified anticommutation relations) there is an arbitrariness in the construction of the Fock space. One could make use of this arbitrariness to choose a basis such that each fermionic field contains only one annihilation and one creation operator (which is equivalent to saying that the $e_\alpha^i(p)$ are indeed the $\delta_{\alpha i}$) and therefore no oscillation phenomenon is produced.

5. Conclusions

We have seen in the previous section that the observations of neutrino oscillations are compatible with their interpretation as a footprint of new physics in the infrared. As far as we are aware, this is the first time that an interpretation of neutrino oscillations coming from new physics, without the need to introduce neutrino masses, and compatible with all experimental results, is presented.

This is achieved because of the indistinguishability of the new mechanism from the conventional one in the range of momenta $p^2 \gg \lambda^2$. In order to reveal the origin of the oscillations it is necessary to go beyond the approximation eq. (4.2), which requires the exploration of the region of small momenta ($p^2 \approx \lambda^2$). To get this result it has been crucial to introduce a new infrared scale through a nonlocal modification of the anticommutation relations of the neutrino field. Gauge invariance forbids a similar nonlocal modification for the remaining fields due to the choice of quantum numbers for the fermion fields in the standard model. In fact the possibility to have the modified anticommutators (2.5) for the neutrino fields is related to the absence of the right-handed neutrino field.

The model of noncanonical fields presented in this work has to be considered only as an example of the general idea that new infrared physics may be present in, or be (partially) responsible of, neutrino oscillations, and that the conventional interpretation may be incomplete. In fact an extension of relativistic quantum field theory based on the modification of canonical anticommutation relations of fields might not be consistent. We have not examined the associated problems of unitarity or causality beyond the free theory. But it seems plausible that the consequences that we have obtained in the neutrino sector will be valid beyond this specific framework.

In conclusion, in this work we have shown that the experimentally observed properties of neutrino oscillations do not necessarily imply the existence of neutrino masses. In fact, future experiments attempting to determine the neutrino mass, such as KATRIN [15], may offer a window to the identification of new physics beyond relativistic quantum field theory in the IR. At this level it is difficult to predict specific observational effects due to the lack of criteria to select a choice for $\tilde{C}_{\alpha\beta}(p)$ in this specific model. A simple example, however, would be the presence of negative eigenvalues of this matrix, which could be reflected in an apparent negative mass squared for the neutrino (see eq. (4.3)) in the fits from the tail of the tritium spectrum. Effects on cosmology could also be possible, again depending on the exact modification of the neutrino dispersion relation in the infrared. All we can say is that if the origin of neutrino oscillations is due to new physics in the infrared then experiments trying to determine the absolute values of neutrino masses and/or cosmological

observations might have a reflection of the generalized energy-momentum relation eq. (3.4) for neutrinos.

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